A rigid-frame structure is a structure made up of linear elements, typically beams and columns, that are connected to one another at their ends with joints that do not allow any relative rotations to occur between the ends of the attached members, although the joints themselves may rotate as a unit. Members are essentially continuous through the joints. As with continuous beams, rigid-frame structures are statically indeterminate.

Many rigid-frame structures resemble simpler post-and-beam systems in appearance, but are radically different in structural behavior, owing to the joint rigidity, which can be sufficient to enable a framed structure to carry significant lateral loads, something a simpler post-and-beam system cannot do without additional bracing elements. Variants of rigid-frame structures have been in use for a long time. A common table, for example, typically derives its stability from the rigid joints that are used to connect the legs to the tabletop. The traditional knee-braced timber structure can also be thought of as a type of rigid-frame structure. Still, the rigid frame as a widely used structural device in major buildings is a relatively recent phenomenon. The development of the steel rigid frame in cities such as Chicago during the latter part of the 19th and early part of the 20th centuries is a major event in the history of structures. The related movement of separating and differentiating enclosure surfaces from the supporting structural skeleton, made possible by the introduction of the rigid frame, also marks a major turning point in the history of architecture. This movement was a marked departure from traditional building practices that made extensive use of dual-functioning elements, such as the exterior load-bearing wall, which served simultaneously as both structure and enclosure.
A useful way to understand how a simple framed structure works is to compare and contrast its behavior under load with a post-and-beam structure that is identical in all respects, except that members in the post-and-beam structure are not rigidly connected, as they are in the framed structure. (See Figure 9.1.)

**Vertical Loads.** A vertical load on a post-and-beam structure is picked up by the horizontal beam and transferred by bending to the columns, which in turn carry the load to the ground. The beam is typically simply supported and merely rests on top of the columns. Consequently, as the vertical load causes bending to develop in the beam, the ends of the beam rotate on the tops of the columns. The angle formed between the beam and a column thus changes slightly with increasing loads on the beam. The columns do not in any way restrain the ends of the beam from rotating. No moment is transferred to the columns, since the beam rotates freely. The columns carry only axial forces.

When a rigid-frame structure is subjected to a vertical load, the load is again picked up by the beam and eventually transferred through the columns to the ground. The load again tends to cause the ends of the beam to rotate. In the frame, however, the column tops and beam ends are rigidly connected. Free rotation at the end of the beam cannot occur. The joint is such that the column tends to prevent or restrain the beam end from rotating. This restraint has several important consequences. One is that the beam now behaves more like a fixed-ended beam than a simply supported one. Thus, the beam has many of the advantages of fixed-ended members discussed in Chapter 8 (e.g., increased rigidity, decreased deflections, and decreased internal bending moments). Nonetheless, the fact that the column top is offering restraint to rotation means that the column must be picking up bending moments in addition to axial forces, thus complicating its design.

The rigid joint does not actually provide full end fixity for either the beam or the column. As the load tends to cause the end of the beam to rotate, the connected top of the column rotates as well. The whole joint between the column and the beam will rotate as a unit. While the whole joint rotates, however, its rigidity causes the members to retain their initial angular relationship to one another (e.g., if the members are initially at 90° to one another, they will remain so). The amount of rotation that occurs depends on the relative stiffnesses of the beam and the column. The stiffer the column relative to the beam, the less total rotation occurs. Some rotation, however, invariably occurs. Thus, the end condition of the beam actually lies somewhere between a fully fixed-ended condition (in which the joint offers full restraint and no rotations occur at all) and a pin connection (in which the member is completely free to rotate). The same is true for the column: Each element enjoys some, but not all, of the advantages of full fixity.

From a member-design viewpoint, the behavior just described generally means that beams in a rigid-frame system carrying vertical loads can be designed to be somewhat smaller than those in a comparable post-and-beam system (since moments are reduced), while the columns may need to be somewhat larger than their post-and-beam counterparts (since they pick up both axial loads and moments, whereas the columns in a post-and-beam system pick up only axial forces). Relative column sizes may be further affected when buckling is considered, because the column in the framed structure has some end restraint, whereas the column in a post-and-beam system has none.
Typical rigid frame structures in steel and concrete.

(a) Post-and-beam structure (beam simply rests on top of column)

The angle formed between column and tangent to the end of the beam changes with changing loads.

Rigid frame structure (beam and column are rigidly joined)

The angle between the tangent to the end of the beam and the tangent to the end of the column remains fixed (in this case at 90°). The whole joint rotates as a unit.

Under lateral loads a post-and-beam structure will collapse unless special precautions are taken (e.g., using shear walls). A frame structure is stable under lateral loads. The beam restrains the top end of each column from freely rotating which would in turn lead to the collapse of the whole structure.

(b) General behavior of rigid frame structures.

Figure 9.1 Types of rigid-frame structures and differences between post-and-beam structures and rigid-frame structures.

Another way that frames are uniquely different from comparable post-and-beam structures in carrying vertical loads discussed before is that frames typically develop horizontal as well as vertical reactions at the ground supports. (Note that this property is intimately related to the presence of the moments in the columns that we discussed before.) The presence of these
(a) The vertical load tends to cause the columns to want to splay outward.

(b) If one of the pinned connections is released, the whole frame would splay horizontally. The application of a horizontal force would tend to cause the structure to resume its original shape. The force required to push the frame exactly back to its original location equals the horizontal thrust normally developed at the same location.

Figure 9.2  Horizontal thrusts in rigid-frame structures carrying vertical loads.

forces can most easily be visualized by imagining the deflected shape the structure would assume if the bases of the columns were not pin connected to the ground foundation and instead were allowed to freely translate horizontally. (See Figure 9.2.) The columns would naturally tend to splay outward. The application of horizontal forces that are inwardly directed at the column bases would cause the bases to tend to resume their original location. The amount of horizontal force that would cause the columns to be forced back into their exact original location equals the amount of horizontal thrust that the frame naturally exerts on the foundation when the column bases are normally attached to it. As discussed later in the chapter, the magnitudes of the moments in the columns and those of the horizontal thrusts are directly related.

The foundations for a frame must be designed to carry the horizontal thrusts generated by the vertical loads. No such horizontal thrusts develop in post-and-beam structures carrying vertical loads. Consequently, foundations can be simpler in post-and-beam systems than in frames.

**Horizontal Loads.** Although the differences in behavior under vertical loads between frame structures and post-and-beam structures are pronounced, the differences with respect to horizontal or lateral loads are enormous. A post-and-beam structure is largely incapable of resisting significant lateral forces. What lateral force resistance such a structure does have stems largely from the large dead weight of the structure when stone or masonry is used or from the presence of some other element, such as an enclosure wall, that provides a bracing function. Column ends in timber structures might be sunk into the ground and thus provide a measure of lateral resistance by virtue of the end fixity that is achieved. By and large, however, most post-and-beam structures are not suited to carrying high lateral forces of the type associated with earthquakes and cyclones.

A rigid-frame structure, by contrast, is well capable of carrying lateral forces if it is designed properly. By virtue of the presence of a rigid connection, the beams restrain the
columns from freely rotating in a way that would lead to the total collapse of the structure. The joints, however, do rotate (to a limited extent) as whole units. The stiffness of the beam contributes to the lateral-load-carrying resistance of a frame, as well as serving to transfer part of the lateral load from one column to the other.

The action of a lateral load on a frame produces bending, shear, and axial forces in all members. Bending moments induced by wind loads are often the highest near the rigid joints. Consequently, members are either made larger or specially reinforced at joints when lateral forces are high.

Rigid frames are applicable to both large and small buildings. Many high-rise buildings use rigid frames to carry both vertical and lateral loads. The higher the building, however, the larger become the forces and moments developed in individual members. The lower columns in a high-rise structure in particular are subjected to very large moments and axial forces because of the huge lateral loads involved. A point is often reached where it becomes unfeasible to design members for these forces and moments, and other bracing systems (e.g., diagonals or shear walls) are introduced to help carry the lateral loads and thus reduce forces and moments in the frames. Even in a lower building, however, supplementary bracing systems are typically used whenever possible, simply because carrying lateral loads by frame action alone is relatively inefficient.

### 9.3.1 Methods of Analysis

**Computer-Based Approaches.** As with continuous beams, rigid-frame structures are statically indeterminate, and their reactions, shears, and moments cannot be determined directly through the application of the basic equations of statics ($\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma M_o = 0$) alone. Because there are typically more unknowns than equations, reactions, shears, and moments are dependent on the precise characteristics of the structure itself, including the relative stiffnesses of beams and columns, as well as the overall geometry of the whole structure and its loading condition.

In current practice, virtually all analyses of rigid-frame structures are done with computer-based analysis programs. Users can input the geometry of overall configurations, specify types of members and support conditions (pinned, fixed, etc.), and specify different types of vertical and horizontal loading patterns. Analysis results include axial forces, shears, bending moments, deflections of joints and members, and other information. Good programs can treat not only two-dimensional frames, but complex three-dimensional configurations as well. The matrix displacement techniques discussed in Appendix 13 frequently form the basis for these computer-based formulations. Finite-element techniques, discussed in Appendix 13, are used as well.

**Approximate Methods of Analysis.** Despite the widespread and correct use of computer-based analysis programs, it remains useful to look at different kinds of approximate methods of analysis that have historically been associated with hand calculations. In this day and age, that is done not so much for their use as practical analysis tools, but because they tend to give the analyst a better sense of how one of these structures is behaving and how forces and moments are generally distributed throughout the structure.
The analytical methods discussed next are based on many simplifying assumptions and yield approximate results that are extremely useful in determining an initial set of member sizes and properties during preliminary design stages. These estimates are then used in more exact analyses made during design development stages, since some member-size estimates must be available before an exact analysis can take place.

An effective method of analysis is based on a procedure that assumes the locations of points of zero internal moment (points of inflection). These locations are initially estimated by looking at the deflected shape of the structure. This is the same basic technique described in Chapter 8 in connection with the approximate analysis of beams.

**Single-Bay Frames: Lateral Loads.** In this section, two simple rigid frames subjected to lateral loads will be analyzed by approximate methods. The first frame is a single-bay frame with rigid connections between the beam and the columns and with pin connections at the column bases. The second frame is identical to the first, except that it has fixed, rather than pinned, connections at the column bases. In both cases, it is assumed that the lateral load carried by the structure can be characterized as a point force acting at the top joint. The exact type and location of the lateral forces carried by a frame in a real building depend on what type of force is operative (e.g., wind or earthquake). An analysis should be made to determine exactly how to model the load. A simple point load will be assumed in the examples that follow.

Figure 9.3 shows the reactions for the first frame. There are four unknown reactions \((R_{A_h}, R_{A_v}, R_{D_h}, \text{ and } R_{D_v})\) and only three equations of statics \((\Sigma F_x = 0, \Sigma F_y = 0, \text{ and } \Sigma M_o = 0)\) available for solution. Consequently, the frame is statically indeterminate to one degree. In this particular frame, it is still possible to determine the vertical reactions, \(R_{A_v}\) and \(R_{D_v}\), simply by summing the moments of the external and reactive forces around either of the pin connections (locations of known zero moment resistance). For the **whole** structure shown in Figure 9.3(a), we have the following equations:

\[
\begin{align*}
\Sigma M_A &= 0: \quad -Ph + R_{A_V}(0) + R_{A_H}(0) + R_{D_V}(L) + R_{D_H}(0) = 0 \\
&\quad \text{so } R_{D_V} = Ph/L \\
\Sigma F_x &= 0: \quad -R_{A_V} + R_{D_V} = 0; \quad \text{or } -R_{A_V} + Ph/L = 0, \quad \text{so } R_{A_V} = Ph/L \\
\Sigma F_y &= 0: \quad +P - R_{A_H} - R_{D_H} = 0; \quad \text{or } R_{A_H} + R_{D_H} = P
\end{align*}
\]

It is evident that the vertical reactive forces can be found only because of the particular condition that the two unknown horizontal reactions pass through the moment center selected. It is not possible to determine any further information by direct application of the equations of statics. It can be surmised that the horizontal reactions, \(R_{A_H}\) and \(R_{D_H}\), are equal, but this is an assumption only.

To get a complete solution to the problem, recall that a point of inflection develops in the beam when the frame carries a horizontal load. By sketching the probable deflected shape of the structure, the location of this point of inflection can be accurately estimated. In this case, the point of inflection is estimated to be at midspan. [See Figure 9.3(a).] The knowledge that, at points of inflection, the internal moment in the structure is zero can be used to provide another independent equation which will lead to a complete solution. For analytical purposes, stipulating that the internal moment is zero at these points is equivalent to inserting a pin connection at that location. The resultant structure can thus be modeled as a statically determinate three-hinged assembly of the type discussed earlier. (See Section 5.5.4.) By decomposing the structure into two separate assemblies at the point of inflection, the horizontal reactions can be determined by
(a) Deflected shape of frame.

(b) Free-body diagrams for parts of frame separated at points of inflection (points of known zero moment). Since the structure is separated at points of zero moment, no internal moments need be shown or considered.

(c) Free-body diagram of individual beam, column, and joint elements. Since the structure is not separated at points of zero moment, the free-body diagrams must show internal moments.

(d) Final moment diagram. Shear and axial forces are present in the members as well as the moment indicated.

Figure 9.3  Simplified analysis of a single-bay rigid frame carrying a lateral load.
considering the moment equilibrium for each piece. [See Figure 9.3(b).] Thus, for the left assembly, \( \Sigma M_N = 0: P(0) + R_{AV}(L/2) - R_{AH}(h) = 0 \). So \( (Ph/L)(L/2) = R_{AH}(h) \) and \( R_{AH} = P/2 \). By considering the horizontal force equilibrium of the whole structure, the remaining unknown reaction, \( R_{DH} \), can be found. For the whole structure \( \Sigma F_x = 0: R_{AH} + R_{DH} = P \) (from before). So \( P/2 + R_{DH} = P \). Thus, \( R_{DH} = P/2 \).

All reactions are now known \( (R_{AH} = P/2, R_{AV} = Ph/L, R_{DH} = P/2, \text{and } R_{DV} = Ph/L) \). Note that the assumption that the point of inflection in the beam was located at midspan is equivalent to assuming that the horizontal reactions are equal. Once the reactions are known, the shears \( V \), moments \( M \), and axial forces \( N \) in the structure can be found by considering each element in turn. [See the free-body diagrams in Figure 9.3(b).] The typical designation system used is as follows: \( M_{xy} \) = moment in member \( x - y \) at the end of the member that frames into joint \( x \). Shears and axial forces are similarly denoted.

Shear forces and normal (or axial) forces are calculated by considering the transverse equilibrium of each section. In this case, they can be determined by inspection. For example, \( V_{BC} = Ph/L \) from \( \Sigma F_y = 0 \). Moments are calculated by multiplying the shear force present by the effective length of the member. For example, \( M_{BC} = (Ph/L)(L/2) = Ph/2 \). Each member is thus treated like a cantilever beam with a concentrated load (in this case, the shear force) at its end.] Results are shown in Figure 9.3. Note that the column on the right is in compression and the one on the left is in tension, but both have similar bending moments. The center beam is in compression, with a positive moment on one end and a negative moment on the other, which in turn give the originally assumed S shape to the member.

The beam moments just found can also be determined by a slightly different procedure making use of another set of free-body diagrams. The free-body diagrams shown in Figure 9.3(c) indicate how the same structure can be broken down into individual beam, column, and joint elements. The concept of isolating joints in a frame and considering their equilibrium is similar to the method of joints used in truss analysis. The primary distinction is that, in trusses, the members were pinned together at joints; hence, moments could not be developed, and only translational equilibrium needed to be considered. In the case of a frame, the member connections are rigid; hence, internal moments are developed at member ends, and those moments must be reflected in the free-body diagrams of individual joints. Because of these moments, both translational and rotational equilibrium need to be considered for any individual joint. With this type of free-body diagram, moments can be found.

As before, a moment is developed at the top of column \( B-A \) because of the horizontal reaction. For column \( B-A \), \( M_{BA} = (P/2)h = Ph/2 \). There is an equal and opposite moment acting on joint \( B \). For rotational equilibrium for the joint to be maintained, a balancing moment must be developed in \( B-C \). The beam provides this moment restraint. At joint \( B \), \( -M_{BA} + M_{BC} = 0 \), so \( M_{BC} = Ph/2 \). Similar observations can be made about column \( C-D \) and joint \( D \). Thus, for column \( C-D \), \( M_{CD} = Ph/2 \), and at joint \( C \), \( -M_{CD} + M_{CB} = 0 \), so \( M_{CB} = Ph/2 \).

The end moments found for the beam are clearly the same as those previously found. Figure 9.3(c) shows not only how the various beam, column, and joint elements are in moment equilibrium, but also how they are in equilibrium with respect to shear and axial forces.

Moment diagrams can now be drawn for each of the beams and columns in the frame. In order to plot these moment diagrams, however, a sign convention other than the one used for horizontal members must be used, since the notion of top and bottom surfaces is meaningless in a vertical member. Common practice in plotting moment diagrams for vertical members is to look at the member from the right and employ the usual convention. (This is the same as turning the member 90° in a clockwise direction.) An equivalent convention that is perhaps more
generally useful, since it is applicable to any member having any orientation, is simply to plot the moment diagram on the compression face of the member and not to be concerned with whether the moment is called a positive one or a negative one.

A more complex type of frame is illustrated in Figure 9.4(a). This frame is identical to the one studied previously, except that the column bases are fixed rather than pinned. The figure shows the reactive forces and moments developed at the foundations. There are six unknown quantities \( R_{A_H}, R_{A_V}, M_{A}, R_{D_H}, R_{D_V}, \) and \( M_{D} \) and only three equations of statics available for use. Hence, the frame is statically indeterminate to the third degree. Consequently, three assumptions must be made if a static analysis is to be used. As before, a sketch of the probably delected shape of the structure is made. As is evident, three points of inflection develop. One is in the midspan of the beam, as before. Two others are near the midheights of the columns. They are not exactly at midheight, since the top joints rotate slightly. If the joints simply translated horizontally without rotation, the points of inflection would have to be identically at midheight (by a symmetry argument). The slight rotation of the upper joints causes the point of inflection to rise somewhat to the location indicated in the figure.

Fixing the location of the three points of inflection makes a static analysis possible. The frame can be decomposed or separated at these points of zero moment in a manner similar to that previously described and each piece treated in turn (i.e., axial forces and shears can be found from the knowledge that the net rotational moments around these points for any piece must be zero). This analysis is illustrated in Figure 9.4(b).

Note that if the frame is decomposed into two sections at the inflection points in the column, the upper part is quite analogous to the frame previously discussed (with pin-connected bases), except that column heights differ. The analysis technique is the same. This structure can then be imagined as simply resting on two vertical cantilever elements. The horizontal thrusts associated with the upper part produce moments in the lower elements. A final moment diagram is shown in Figure 9.4(d).

Note that the final moments in individual members are lower than in the pinned frame previously analyzed.

**Single-Bay Frames: Vertical Loads.** The general procedure for making an approximate analysis of a frame carrying vertical loads is much the same as the one described in the previous section for lateral loads. Consider the rigid frame shown in Figure 9.5(a), which has pinned connections at the column bases. The first step in the analysis is to sketch the probable deflected shape of the structure and to locate points of inflection.

It is somewhat more difficult to locate inflection points associated with vertical loads than those associated with lateral loads. If the joints provided full fixity (and no end rotations occurred at all at beam ends), then the points of inflection would be \( 0.21L \) from either end. (See Section 8.3.) Since some rotations do occur, but the joint does not freely rotate, the actual end conditions lie somewhere between pinned connections and fully fixed connections; the inflection points in the beam thus lie somewhere between \( 0L \) and \( 0.21L \) from the joints. For beams and columns of normal stiffnesses, the inflection points are normally found about \( 0.1L \) from either end. As will be discussed more fully in Section 9.3.2, the precise location of the inflection points is highly sensitive to the relative stiffnesses of the beams and columns.

If the inflection points are assumed to be \( 0.1L \) from either end, the structure can be decomposed into three statically determinate elements and analyzed as illustrated in Figure 9.5(b). The final moment diagram obtained is shown in Figure 9.5(c).
(a) Deflected shape of frame.

\[ M_{BC} = (0.45Ph/L)(L/2) = 0.225Ph \]
\[ M_{CB} = (0.45Ph/L)(L/2) = 0.225Ph \]
\[ M_{CD} = (P/2)(0.45h) = 0.225Ph \]
\[ M_{D} = (0.55h)(P/2) = 0.275Ph \]

(b) Free-body diagram for parts of structure separated at points of inflection (points of known zero moment).

(c) Free-body of individual beam column, and joint elements. Each element must be in a state of translatory and rotational equilibrium.

\[ M_{BC} = 0.225Ph \]
\[ M_{BA} = 0.225Ph \]
\[ M_{CS} = 0.225Ph \]
\[ M_{CD} = 0.225Ph \]
\[ M_{DA} = 0.275Ph \]
\[ M_{D} = 0.275Ph \]

(d) Final moment diagram. Shear and axial forces are present in the members as well as the moments indicated.

Figure 9.4 Simplified analysis of a single-bay rigid frame carrying a lateral load.
(a) Frame and loading. Inflection points are developed near both ends of the beams. Their locations are assumed to be as shown.

(b) Free-body diagrams of parts of the frame separated at inflection points. Shears, moments, and axial forces are found in each piece by a static analysis.

\[ M = w(0.8L)^2/8 = 0.08wl^2 \]

\[ M_{CB} = 0.4wl(0.1L) + w(0.1L)^2/2 = 0.045wl^2 \]

\[ \Sigma M_C = 0 \]

\[ 0.4wl(0.1L) + w(0.1L)^2/2 - R_{D_1}(h) = 0 \]

\[ R_{D_1} = 0.045wl^2/h \]

(c) Final moment diagram.

\[ 0.08wl^2 \]

\[ wL^2/8 \]

(d) The moment present is proportional to the deviation of the structure from the funicular line for the loading.

\[ \text{Funicular line for uniformly distributed loading (passes through inflection points)} \]

**Figure 9.5** Simplified analysis of a single-bay rigid frame carrying vertical loads.
As is evident, the vertical loads produce moments both in the beam and in the columns. Maximum moments in the beam normally occur at midspan, but critical moments are present at end joints as well. Maximum column moments invariably occur at member ends.

### 9.3.2 Importance of Relative Beam and Column Stiffnesses

In any statically indeterminate structure, including a frame, the magnitudes of the internal forces and moments are, in the final analysis, dependent on the relative properties of the members themselves. None of the approximate analyses discussed thus far have reflected the importance of this fact. Implicit in the analyses, however, have been assumptions about the relative characteristics of the members used. Normal or typical stiffnesses were considered.

The importance of different member properties can be seen by looking at Figure 9.6. If it is initially assumed that one column is stiffer than the other (i.e., one has a higher relative

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**Figure 9.6** Effects of different column stiffnesses on forces and moments in a rigid structure.
(a) Three-hinged arch structure. Large negative moments are developed in the beam.

(b) Post-and-beam structure. Large positive moments are developed in the beam. The columns do not restrain the rotations of the beam ends at all.

(c) Frame with very flexible columns and an extremely stiff beam. The flexible columns do not provide significant restraint to the ends of the beam. The beam behaves similarly to a pin-ended one.

(d) Frame with normal beam and column stiffness. The columns offer partial restraint to the ends of the beams. Some rotations occur. Negative moments increase and positive moments decrease.

(e) Frame with very stiff columns and a flexible beam. The columns offer almost full restraint to the ends of the more flexible beam.

(f) The sum of the positive and negative beam moments remains the same in each case. The location of the base line and hence of the design moments varies.

Figure 9.7 Effects of different relative beam and column stiffnesses on internal forces and moments in a rigid-frame structure. The moments generated in the frames by the vertical load vary with different locations of the inflection points. The stiffer the section of a frame, the greater is the moment developed at the section.

$I$ value), the stiffer column will end up taking a greater share of the horizontal load than the more flexible one will. An assumption that horizontal reactions are equal is thus not tenable. Higher moments would also be developed in the stiffer member rather than in the more flexible one, as a consequence of the former’s taking a greater portion of the load.

Different relative stiffnesses between beams and columns also affect moments due to vertical loads. As Figure 9.7 illustrates, the location of the inflection points are affected by the relative stiffness of the beams and columns. The stiffer the column relative to the beam, the more
it restrains the end of the beam from rotating. Consequently, higher moments are developed in
the column when it is very stiff relative to the beam, rather than when the column is more
flexible. Negative moments in the beam are also increased, whereas positive moments are
decreased. When the beam is stiff relative to the column, converse phenomena occur.

Exact methods of analysis are available that take into account these effects. Such analyses
must be based on an assumed set of beam-and-column stiffnesses. The approximate
analyses previously discussed are useful in providing these initial estimates.

EXAMPLE

Crown Hall in Chicago, by Mies Van der Rohe (Figure 9.8), is a well-known example of the use of
exposed rigid frames. The horizontal members are considerably stiffer than the supporting columns, so it
can be expected that relative stiffness issues are extremely important. Determine the bending moments
present in a typical interior frame. All member properties, dimensions, etc., are estimated. The base con-
nection is, in actuality, a complicated connection involving two pins, one at the base of each column and
another just above it. Use any available computer-based structural analysis program.

Figure 9.8  The moment distribution illustrates the importance of relative stiffness considerations. The
values obtained are quite different from those obtained by estimating points of inflection and using hand
calculations.

Solution: See Figure 9.8.

Note:
The moment distribution obtained illustrates the importance of relative beam-to-column stiffness consid-
erations. The locations of the points of inflection are different from those previously estimated (which
were based on other beam-to-column stiffness ratios). The moment values obtained are correspondingly
quite different from those which would be obtained by using hand calculations.
9.3.3 Sidesway

A phenomenon of particular interest in frames carrying vertical loads is sidesway. If a frame is not symmetrically shaped and loaded, the structure will sway (translate horizontally) to one side or the other. The reason for sidesway can be seen by inspecting Figure 9.9. Assume that the columns are very stiff and completely restrain the ends of the beam, which can then be modeled as a fixed-ended beam. Corresponding fixed-ended moments are shown in Figure 9.9(b). Since the joints must be in rotational equilibrium, the moments at the column tops are of the same magnitude as those in the adjacent beam. For equilibrium of the column, the presence of a moment at the top implies the necessity of a horizontal thrust at the base of the frame. The magnitude of this thrust at the base of a column is directly related to the magnitude of the moment at the top. Since the moments are unequal, the thrusts are also unequal. By looking at the overall equilibrium of the frame in the horizontal direction, however, it can be seen that the thrusts must be equal, so that \( \Sigma F_x = 0 \) obtains, which in turn means that the moments at the column tops and beam ends must also be equal. The only way that such an equality can be naturally obtained is for the frame to sway to the left. As it sways to the left, there will be a tendency for the upper right joint to open up slightly, thus reducing the moment that is present (and the thrust of the column base), and for the upper left joint to close up slightly, thus increasing the moment that is present (and the thrust at the column base). Just enough sway will occur so that moment and horizontal thrust will be equal.

9.3.4 Support Settlements

As with continuous beams, rigid frames are quite sensitive to differential support settlements. Consider the rigid frame shown in Figure 9.10(a). Any type of differential support movement, either vertically or horizontally, will induce moments in the frame. The greater the differential settlements, the greater the moments induced. If not anticipated, these moments can lead to failures in the design of the frame. For this reason, special care must be taken with the design of foundations for rigid-frame structures.

9.3.5 Effects of Partial-Loading Conditions

As with continuous beams, the maximum moments developed in a frame often occur, not when the frame is fully loaded, but when it is only partially loaded. This, of course, complicates the analysis process enormously. The first problem is simply that of predicting which type of loading pattern produces the most critical moments. Only after that is done can an analysis take place. Usually, some variant of a checkerboard loading pattern over the whole frame produces maximum positive or negative moments at the location considered. There are several techniques (e.g., the Miller-Breslau technique) for establishing exactly which loading conditions are most critical. Going into the problem of partial loadings in detail, however, is beyond the scope of this book.

9.3.6 Multistory Frames

As previously mentioned, virtually all analyses of rigid-frame structures are currently done with computer-based analysis programs. This is particularly true for complex multistory structures. These formulations allow shears, bending moments, and axial forces to be obtained for each member in a frame. The need to check different types of critical loading patterns is fully facilitated by
(a) A rigid frame that is not symmetrically loaded.

(b) Frame moments assuming that no joint rotation at all occurs.

(c) The assumption that no joint rotation at all occurs leads to the result that horizontal frame reactions at the foundation are unequal.

(d) From a consideration of the overall equilibrium of the frame in the horizontal direction, it is evident that the result found in (c) is not plausible. Horizontal reactions must be equal. This also implies that moments at the column tops must also be equal.

(e) The only way that the frame can naturally adjust itself so that moments at column tops are equal (so that horizontal reactions are equal) is to sway to the left. This has a tendency of closing up the upper left joint (and increasing the moments present) and opening up the upper right joint (and decreasing the moments present). The frame will sway until the moments present become equal.

Figure 9.9 Sidesway in rigid frames. The absence of complete symmetry in a frame or its loading leads to a horizontal sway in the structure.
Figure 9.10 Effects of support settlements on rigid-frame structures. Curvatures, and hence bending moments, are induced in a frame because of differential joint settlement.

The computer programs. Nonetheless, it is still interesting to look at hand-calculation techniques that were developed to analyze multistory frames, because they tend to give the analyst a better sense of how forces and moments are generally distributed throughout the structure.

Several approximate methods are used to analyze multistory frames subjected to lateral forces. A time-honored approach called the cantilever method was first introduced in 1908. It involves making many of the same types of assumptions previously made, plus others. Generally, the technique assumes the following: (1) There is a point of inflection at the midspan of each beam in a complex frame; (2) there is a point of inflection at the midheight of each column; and (3) the magnitude of the axial force present in each column of a story is proportional to the horizontal distance of that column from the centroid of all the columns of the story.

These assumptions are diagrammatically illustrated in Figure 9.11. The illustrations also provide an insight into the behavior of a multistory frame under load. Moments are generated in all members. The magnitudes of these moments depend upon the magnitude of the resultant shear force \( V_L \) (the sum of the lateral loads above the floor considered). This shear force is balanced by resisting shear forces \( (V_{C1}, V_{C2}, \ldots, V_{Cn}) \) developed in the columns at the same level \( (V_L = \Sigma V_{Cn}) \). The moment in any column is the shear force in the column, multiplied by its moment arm (one-half the column height). Moments in beams are generated to balance moments at column ends. Since the total shear force \( V_L \) is greater at lower floors than at upper floors, bending moments in beams and columns are greater at lower rather than upper floors.

The magnitudes of the axial forces in the columns depend on the magnitude of the overturning moment \( M_L \) associated with the lateral loads above the section considered. Consequently, axial forces are greater at the base of the building, where the overturning moment is the greatest; they tend to diminish in upper-story columns.

The magnitudes of forces and moments in multistory frames that are due to vertical loads can be estimated in the same way as was illustrated for single-bay rigid frames. Inflection points can be assumed at 0.1\( L \) from either end of the beam. This assumption has the effect of
Wind acting against the face of the building creates overturning moments and sliding forces. Reactions developed at the base provide resisting forces and moments.

Lower-story columns: The wind force causes the columns to rack laterally. Curvatures and related internal forces and moments are developed in columns and beams.

Bending moments

A typical column is subjected to axial forces, shears, and bending moments. Beams are also subjected to similar forces.

Figure 9.11  Multistory frames.
creating a statically determinate beam between the two inflection points that is supported by short cantilevers. Positive and negative moments can then be found by statics.

Other, more exact methods of analysis that are computer oriented are now more commonly used to analyze multistory frames. (See Appendix 13.) Nonetheless, the cantilever method still provides a useful way of conceptualizing the behavior of multistory structures.

9.3.7 Vierendeel Frames

The discussion thus far has dealt with frames that are used vertically. Frames can also be used horizontally, as illustrated by the Vierendeel structure. (See Figure 9.12.) This structure resembles a parallel-chord truss with the diagonals removed and is often used in much the same way as a truss. The structure is used in buildings whose functional requirements mitigate against the presence of diagonals. The structure is considerably less efficient, however, than a comparable structure having diagonals.

Vierendeel structures are analyzed in much the same way as previously described for vertical frames. Locations of inflection points are estimated and used to provide sufficient information to determine internal shears, moments, and axial forces.

The internal moments in elements tend to be the highest in members near the end of the structure and lower toward the middle. This distribution reflects the fact that the overall shear forces associated with the total loading, which cause local bending moments and shears in specific members, are highest toward the ends and decrease toward the middle. Axial forces in members are highest in middle top and bottom chord elements and decrease toward the ends. This distribution reflects that of the overall moment associated with the total loading. The pattern is similar to that present in parallel-chord trusses.

The results of a finite-element analysis (see Section 6.3.9 and Appendix 14) for a typical Vierendeel structure are given in Figure 9.13. The deformation analysis clearly shows the reverse curves developed in typical members. These reverse curves are associated with bending stresses that not only vary in magnitude, but also change from tension to compression along the length of a member.

9.4.1 Introduction

Designing a frame structure can be an involved process. One of the first questions to be asked is simply whether it makes sense to use frames at all. They are not, for example, particularly efficient types of structures to be used in situations where lateral loading conditions are high. Simply because frames can be designed to carry lateral loads does not mean they should be given preference over other approaches, such as structures using shear walls or diagonal bracing, that are better suited for carrying this type of loading. Frames make sense when the requirements of a building do not easily allow other solutions to be used.

When frames are warranted, the general geometry and dimensions of the frame to be designed are often fixed in the building context, and the design problem frequently becomes a more limited one of developing strategies for the selection of connection types, selecting materials, and sizing members.