Chapter Eight

Frames and Arches
8.1 Post and Lintel

From time immemorial the problem of sheltering human beings from the weather has been solved by an enclosure of walls topped by a roof. In prehistoric times walls and roof were made of the same material, without any distinction between a supporting "structure" and the protecting "skin." A separation of the supporting and protecting functions leads to the simplest "framed" system: the post and lintel (Fig. 8.1).

The lintel is a beam simply supported on two posts and carrying the roof load. The posts are vertical struts compressed by the lintel. The posts must also resist some horizontal loads, such as the wind forces; this resistance stems from a bending capacity in the case of wooden or steel posts, from their own weight in stone or masonry piers. Some connection between post and lintel must also be provided, lest the wind blow the roof away.

The foundations of the posts carry the roof and post loads to the ground by means of footings which spread the load and guarantee that soil settlements will be limited (Fig. 8.1). In any case, the posts and the foundations are essentially under compression, and this is characteristic of the post-and-lintel system.

Post-and-lintel systems may be built one on top of another to frame multistory buildings. In this case the lintels are supported by vertical columns, or bearing walls of stone or masonry as high as the entire building (Fig. 8.2). Construction of this type, while capable of carrying vertical loads, is not well suited to resist horizontal loads, and is easily damaged by hurricane winds and earthquakes. This happens because masonry or stone elements have little bending resistance, and a strong connection between the horizontal and the vertical structural elements is not easily developed.
8.2 The Simple Frame

The action of the post-and-lintel system changes substantially if a rigid connection is developed between the lintel and bending-resistant posts. This new structure, the simple or single-bay frame, behaves monolithically, and is stronger than the post-and-lintel against both vertical and horizontal loads (Fig. 8.3).

Under a uniform load the lintel of a post-and-lintel system deflects, and its ends rotate freely with respect to the posts, which remain vertical. In order to grasp the action of the rigid frame under the same load, one may first consider the horizontal beam as simply supported at its ends, and the columns as rigidly connected to the rotated ends of the beam, so as to stick out in a straight, inclined position [Fig. 8.4(a)]. In order to bring back the feet of the columns to their supports, the columns must be forced inward by horizontal forces, and the ends of the beam must rotate partially backwards. In their final position both the beam and the columns of the frame are curved and develop bending stresses [Fig. 8.4(b)]. The beam has partially restrained ends, and behaves like the center span of a continuous beam on four supports [see Fig. 7.35(b)].
8.6 (a) Stresses due to compression and bending in right column of simple frame.

All three members of a simple frame under vertical loads are bent and compressed. Simple bending develops a linear stress distribution across the depth of the element, with maximum tensile and compressive stresses of equal value; compression adds a constant compressive stress, which develops the resultant trapezoidal stress distributions shown in Fig. 8.6. Usually compression prevails in the columns so that their stresses are entirely compressive [Fig. 8.6(a)]; bending prevails in the beam, so that tensile stresses are developed in some parts of the beam [Fig. 8.6(b)].

The foot of the column may be either "hinged" or "fixed." In steel frames the first type of connection may be provided by an actual hinge [Fig. 8.7(a)] or by suitably located anchor bolts; in reinforced-concrete frames it is provided by crossing the reinforcing bars so as to reduce the bending capacity of the section [Fig. 8.7(b)]. The deformation of the frame under vertical load shows that a hinged frame usually has smaller compressive stresses on the outer column fibers than on the inner fibers [Fig. 8.6(b)].
The deformations of a fixed frame under vertical loads show that the columns develop inflection points (Fig. 8.8). Since an inflection point is a point of no curvature, it is equivalent to a hinge where no bending stresses are developed. Hence, the fixed frame is roughly equivalent to a hinged frame with shorter columns and is stiffer than the hinged frame. The thrust in the fixed frame is greater than in the hinged frame, since it takes a larger force to bring back (even if partially) its shorter, stiffer equivalent columns.

Frames are stronger against vertical loads than post-and-lintel systems, but their action is even more advantageous in resisting lateral loads. A post loaded by a horizontal load, such as the wind pressure, acts as a single cantilevered beam without any collaboration from the lintel or the opposite post [Fig. 8.9(a)]. In the frame, instead, continuity with the beam transfers part of the wind load to the opposite column, as shown by the fact that both columns bend [Fig. 8.9(b)]. Even if the beam were hinged at the tops of the columns, because of its rigidity in compression it would carry half the wind load to the leeward column, and cut in half the bending stresses in the columns (Fig. 8.10). Since, moreover, the rigidity of the frame connections compels the beam to bend together with the columns, an additional restraint is introduced in the columns, which become stiffer [Fig. 8.9(b)]. Their deflection is reduced, and so are their bending stresses.
The tendency of the frame to turn over due to the pressure of the wind is balanced by additional reactions in the columns: tensile in the windward column and compressive in the leeward column [see Fig. 8.9(b)]; but these forces are usually small because their lever arm, equal to the width of the frame, is large compared to the lever arm of the wind pressure, which is half the frame height. The lateral displacement or sidesway produced in a frame by lateral loads is also present when unsymmetrical vertical loads are applied to the frame (Fig. 8.11).

Whenever the beam of a fixed frame is much stiffer in bending than the columns, under lateral or unsymmetrical loads the inflection points in the columns appear approximately at midheight; the shortened legs of the equivalent hinged frame show that the fixed frame is stiffer than the hinged frame against lateral loads also (Fig. 8.12). In either type of frame lateral loads must be considered acting from either side of the frame, so that the reinforcement of concrete frames must be located along both the inner and the outer fibers of the posts, and the top and bottom fibers of the beam.

The bending-strength requirements of the columns are often so small as to make the entire frame flexible. If for functional reasons an open bay is required, a substantial increase in lateral rigidity of the frame can be achieved only by increasing the bending rigidity of its columns. On the other hand, lateral rigidity is inexpensively obtained by tensile or compressive elements; hence, tensile or compressive diagonals may be used to stiffen the frame with little increase in material (Fig. 8.13) and with a large reduction of bending stresses in the columns and beam. In general, structural systems with rectangular meshes, such as frames, are more flexible than triangulated systems, but triangulated systems seldom meet the functional requirements of a modern building.
8.3 Multiple Frames

The advantages of continuity can be compounded by the use of multiple frames, in which a horizontal beam is supported by, and rigidly connected to, three or more columns.

In view of the rigidity of compressed elements (see Section 5.2), the vertical deflections of the columns of a multiple frame are small, and the behavior of its beam, when loaded vertically, is similar to, although not identical with, that of a continuous beam on rigid supports (see Section 7.3). Under the action of loads concentrated on one span the multiple frame develops curvature not only in all other spans, as a continuous beam, but in all columns as well. The frame as a whole also exhibits a lateral displacement, the so-called sidesway, which is absent in the continuous beam (Fig. 8.14).

If the spans and the vertical loads of the multiple frame do not differ substantially from bay to bay, the thrusts in adjoining frames act in opposite directions and tend to cancel each other. Theoretically only the two outer bays need to be tied or buttressed. In practice, some means must always be provided to take care of uneven loadings on the various spans of the multiple frame: the excess thrust on each column is often resisted by the action of the soil on the foundations.

Multiple frames are efficient in resisting lateral loads. The rigidity of the beam against compressive loads makes the lateral deflections of the tops of all the columns practically identical. Hence, if the columns are identical, the lateral load is carried equally by all the columns of the frame, each acting as a cantilever and additionally stiffened by the bending rigidity of the beam. The overturning tendency due to wind action is equilibrated by tensile reactions on half the frame columns on the windward side, and by compressive reactions on the half on the leeward side. In view of the greater number of column reactions and their large lever arms, these forces are relatively small. Multiple frames in the external walls or in the core walls of a building are often used to carry the wind load on the faces perpendicular to these walls.
A single-bay frame with a beam rigidly connecting the feet of the columns is a closed structural element capable of carrying vertical and horizontal loads [Fig. 8.15(a)]. A multiple-bay frame with continuous beams at the top and the bottom of the columns is also a closed system, and may be used as a truss to span large distances [Fig. 8.15(b)]. In this case, the multiple frame may be thought of as a beam with compression and tension flanges, in which the shear is resisted by the columns. The “secondary” bending stresses in the horizontal and vertical elements of such truss-frames (see Section 7.4) are much greater than in the members of triangulated trusses, but the simplicity of their connections and their unencumbered bays have made these truss-frames popular. Such Vierendeel trusses, named after their Belgian inventor, are commonly used in bridge design. They are also used in buildings when the structure is supported on wide spans and unencumbered bays are essential. In this case, the truss consists actually of the columns and floor beams of the building (Fig. 8.16). Multistory frames are commonly used in tall structures. Their action against vertical loads is similar to that of single-bay frames, with the added advantage that the horizontal beams act both as load-carrying elements for a given floor and as tie-rods for the frame above it.
The action of a single-bay, multi-story frame under lateral loads is similar to the action of a cantilevered Vierendeel truss: the leeward columns are in compression and bending, the windward columns are in tension and bending, while the floor beams or slabs transmit the shear from the tension to the compression elements (Fig. 8.17). One may also think of a multi-bay, multi-story frame as a gigantic I beam, cantilevered from the ground, in which the depth equals the width of the frame and the floor beams act as a discontinuous web. The continuity between the floor beams and the columns makes the entire structure monolithic and introduces bending in both the horizontal and the vertical elements. A concentrated vertical load is felt in the entire structure, each part of which collaborates in carrying it (Fig. 8.18).

As the height and the width of the building increase, it becomes practical to increase the number of bays so as to reduce the beam spans and to absorb horizontal loads more economically. The resisting structure of the building thus becomes a frame with a number of rectangular meshes, allowing free circulation inside the building, and capable of resisting both vertical and horizontal loads. A number of such frames, parallel to each other and connected by horizontal beams, constitute the cage structure encountered in most steel or concrete buildings today. These three-dimensional frames act integrally against horizontal loads coming from any direction, since their columns may be considered as part of either system of frames at right angles to each other (Fig. 8.19).
The high-rise building, or skyscraper, is one of the great conquests of modern structural design, made possible by the multi-story frame and the high strength of steel and concrete. The Empire State Building in New York, built in 1930, has 102 floors and is 1050 feet high, not including the 200-foot upper tower originally designed to anchor dirigibles, and the 222-foot TV antenna. The John Hancock Insurance Company building in Chicago (Fig. 8.20), the twin towers of the World Trade Center in New York, and the Sears building also in Chicago (Fig. 8.21), the tallest building in the world to date (1986), all built in the 1970s, have heights of between 1200 and 1450 feet and steel structures consisting of frames with bays of narrow width on the outside of the building. Such structures behave essentially as cantilevered steel tubes of rectangular cross section. Tubular high-rise steel buildings have also been built by "bundling" a number of tubular frames and with other than rectangular cross section, e.g., triangular.
Reinforced concrete skyscrapers cannot reach the height of steel skyscrapers, but have been built to heights of over 800 feet; the tallest to date (1986) is the 895-foot Water Tower Building in Chicago. They usually consist of outer frames and of an inner core built by means of concrete walls. The lateral stiffness of the boxed core is usually so large relative to that of the outer frame that the lateral forces due to wind or earthquake are predominantly resisted by the so-called shear walls of the core. (These walls resist the horizontal loads as cantilevered beams in bending; their misleading name indicates that they resist the wind shear.)

Because of their lateral stiffness, concrete core walls are also used in high-rise buildings with outer steel frames (Fig. 8.22), thus combining the advantages of smaller steel columns to resist vertical loads only and of concrete walls to resist lateral loads. The steel frame of the core of a steel building may also be stiffened by steel diagonals or by prefabricated concrete panels inserted in its bays.

A skyscraper is nothing but a slender, vertical, cantilevered beam resisting lateral and vertical loads. The vertical floor loads accumulate from the top to the bottom of the building, requiring larger columns as one moves down. Were it not for the high compressive strength of modern structural materials, the area of the columns would encroach on the usable floor area to the point where this might become nil.
The efficiency of frame and shear-wall action is required by skyscrapers to resist large horizontal wind loads with minor deflections. A constant wind pressure of 46 pounds per square foot (due to a 113 miles per hour wind) represents a total load of 12 million 500 thousand pounds on the 1350 foot cantilever of one of the World Trade Center Towers in New York; under the action of this wind load the top of the building sways approximately three feet each side of vertical. In order to avoid discomfort to the occupants of the top floors of a high-rise building, the top deflection or drift must be limited to between one thousandth and one five-hundredth of its height. Such stiffness is required because human discomfort is severe when the period of the swaying is in resonance with the period of the human insides.

The columns, beams, and walls of a framed structure are its resisting "skeleton." In order to enclose the space defined by the skeleton and to make it usable, the exterior of the building is covered with a "skin" and the floor areas are spanned by horizontal floor systems. The skin of modern buildings is often made of metal or concrete, and glass, and called a curtain wall. A curtain wall of prefabricated concrete or brick panels may also be used to enclose buildings and must be connected to the structure so as to allow expansion and contraction of the panels under differences in temperature between the exterior and the interior of the buildings. The floor structure consists of long-span beams connecting the outer frames to the core, of secondary beams or joists spanning the distance between the main beams, and of slabs of concrete or steel decks spanning the distance between the secondary beams.

Floor systems are analyzed in Chapter 10.

8.4 Gabled Frames and Arches

All three members of a single-bay frame under vertical loads are subjected to compressive and bending stresses. The columns are compressed by the loads on the beam and bent by the rotation of the joint connecting them rigidly to the beam. The beam is bent by the loads on it and compressed by the thrust from the foot of the columns. With the usual proportions of beam and columns, compression prevails in the columns and bending in the beams, since the columns are relatively slender and the beam is relatively deep.